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# Precision Electroweak Constraints on RS Grand Unification

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## Outline:

1. Introduction and Motivation
2. RS Unification
3. Phenomenology and Precision EW Fit
4. Outlook

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## Introduction: Why Warped Extra Dimensions?

- The Randall-Sundrum model is a 5d model in a slice of  $\text{AdS}_5$ :

$$dX^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- The curvature is  $k \sim M_{Pl}$ .
- An exponential “warp factor” red-shifts the Higgs VEV (on the IR brane) from  $M_{Pl}$  to  $M_W$ .

$$\begin{aligned} & \sqrt{R} \left[ g^{\mu\nu} (D_\mu H)^\dagger (D_\nu H) + \lambda (|H|^2 - v^2)^2 \right] \\ & \rightarrow (D_\mu \tilde{H})^\dagger (D^\mu \tilde{H}) + \lambda \left( |\tilde{H}|^2 - e^{-2kL} v^2 \right)^2 \end{aligned}$$

- If  $v \sim k \sim M_{Pl}$ ,  $v e^{-kL}$  can still be  $\mathcal{O}(246 \text{ GeV})$  provided  $kL \sim 30$ .
- We will generalize to gauge fields in the bulk.
- The hierarchy problem solution really only demands that the Higgs itself be localized on the TeV brane.

## Why Bulk Gauge Fields in RS?

- From a field theory point of view, gauge field localization is not understood.
- ADS/CFT suggests that this 5d theory is dual to a 4d theory with a composite Higgs.
- It has been shown that the low energy couplings of bulk gauge fields show a log dependence on the fundamental parameters:

$$\frac{1}{g^2(q)} = \frac{L}{g_5^2} + \frac{1}{g_{UV}^2} + \frac{1}{g_{IR}^2} + \frac{b}{8\pi^2} \log \frac{k}{q} + \Delta_i kL + \mathcal{O}(1).$$

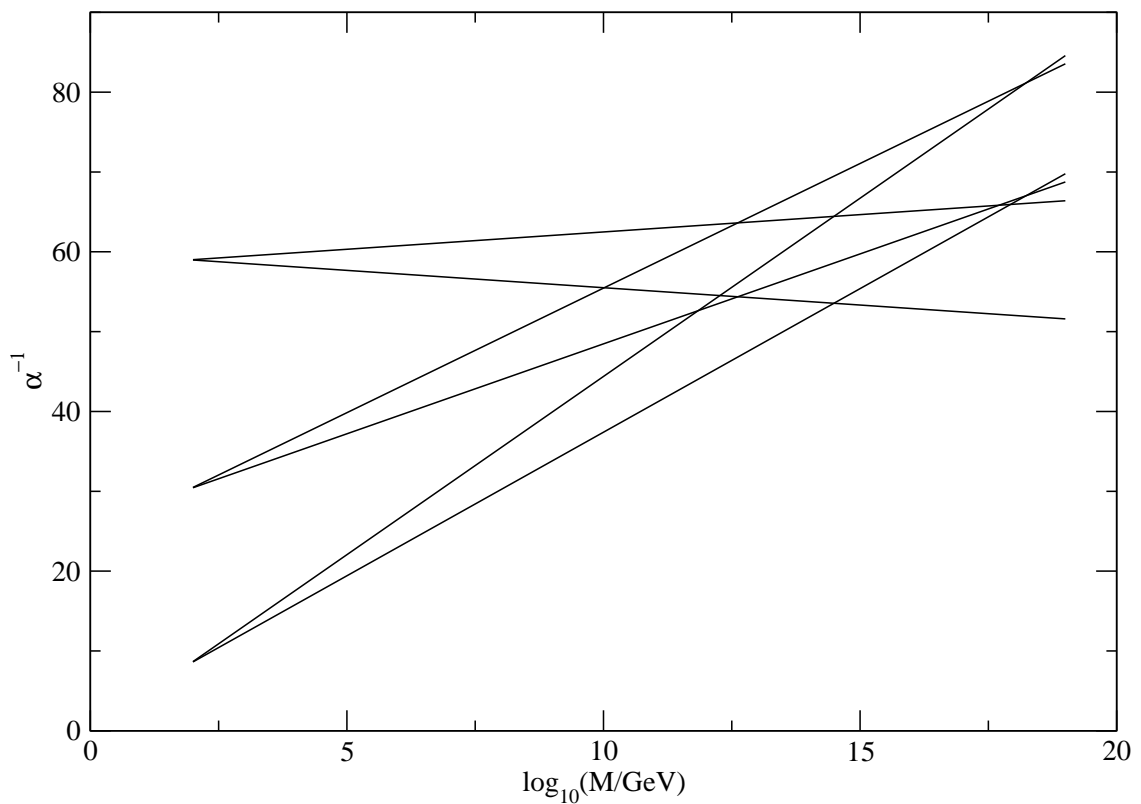
- The log dependence can be understood from the fact that KK modes couple weakly to the UV brane.
- In fact, the log is a renormalization of the UV brane kinetic term.
- Above  $k$ , couplings show power-law dependence, as in flat extra dimensions.
- I will assume the GUT breaking is order  $k$ , so that piece is unified.

# RS Grand Unification

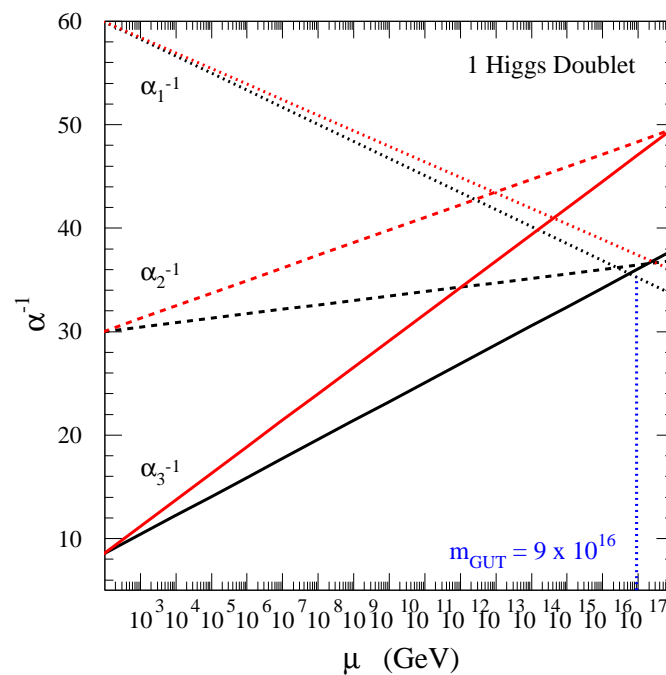
- $\Delta_i$  represents bulk GUT-breaking threshold effects.
- With bulk gauge fields, the  $\log$  can be as large as the one typically inferred from a 4d GUT: Grand Unification is a possibility.
- This has been used by Agashe, Delgado, and Sundrum to construct non-SUSY GUTs which use RS to address the hierarchy problem.
- They invoke split multiplets to prevent TeV scale proton decay.
- So unlike the traditional SU(5) GUT, this model does not have truly unified matter.
- I will be interested in the precision EW constraints on this theory.
- This will allow us to see how well the model is able to accomplish what it sets out to do.
- In particular, we can see if  $v \lesssim k$  is allowed.

## Couplings Unified?

- The KK modes form complete GUT multiplets, so the zero modes determine whether the couplings unify.
- The **ADS** model invokes bulk GUT-breaking to provide large threshold corrections.
- Because it is a bulk effect, it is enhanced compared to the **log** dependence of the UV brane coupling.



- An elegant alternative is to invoke split multiplets of vector-like quarks.
- This has the advantage that it only depends on the gauge assignment of the spectator fermions.



- This matter content ( $Q, \bar{Q}, d, \bar{d}$ ) was motivated to address  $A_{FB}^b$ .
- We will be more concerned with the low energy theory - insensitive to how precisely coupling unification is achieved.

# Framework

- We will use the general features to inspire our study.
- We imagine an SU(5) GUT in the bulk, with the gauge symmetry broken by boundary conditions.  
[ i.e. : Hall, Nomura ]
- It will be useful to have this occur on the IR brane.
- The theory has **no SUSY**; the RS mechanism addresses the hierarchy problem.
- Fermions will be in the bulk, composed of split multiplets.
- This is motivated by EW data, proton decay, flavor physics.
- The UV brane kinetic terms are responsible for differences of low energy couplings: log terms.
- IR brane kinetic terms are dangerous: destroy unification if large.

# KK Decomposition: Gauge Fields

[Pomarol; Davoudiasl, Hewett, Rizzo]

- We expand vector fields in KK modes:

$$V_\mu(x^\mu, y) = \sum_n f^n(y) V_\mu^n(x^\mu)$$

- In a warped background, we have the eigenvalue equation:

$$[\partial_y^2 - 2k\partial_y + e^{2ky} m_n^2] f_n(y) = 0$$

- The solutions are Bessel Functions:

$$e^{k|y|} \left\{ \mathbf{J}_1 \left( \frac{m_n}{k} e^{k|y|} \right) + b \mathbf{Y}_1 \left( \frac{m_n}{k} e^{k|y|} \right) \right\}$$

- With quantized Masses:

$$J_0 \left( \frac{m_n}{k} \right) Y_0 \left( \frac{m_n}{k} e^{kL} \right) = J_0 \left( \frac{m_n}{k} e^{kL} \right) Y_0 \left( \frac{m_n}{k} \right)$$

- Though the dimension itself is small, the KK spectrum is TeV-spaced  $\sim k e^{-kL}$ .

- KK modes couple to the IR brane:  $\sqrt{2kL} \times g_0$ .

# Localized Gauge Kinetic Terms

- A 5d theory with bulk gauge fields:

$$-\frac{1}{4g_5^2} \mathcal{F}^{MN} \mathcal{F}_{MN} - \delta(x_5) \frac{1}{4g_a^2} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}$$

- $\mathcal{F}^{MN}$  is the gauge field strength tensor
- $g_5$  is the dimension -1/2 bulk gauge coupling.
- $g_a$  is the dimensionless gauge coupling on the brane
- As we will see, the local term expels the KK modes of the gauge fields, thus making the brane seem opaque.
- After rescaling  $g_5$  into  $\mathcal{A}^M$ , the brane term is  $r_c \equiv g_5^2/g_a^2$  ( $r_c$  has dimensions of length)
- $r_c \rightarrow 0$  recovers the usual “transparent brane” case
- UV brane kinetic terms: mostly redefine couplings.
- IR brane kinetic terms: large effect on wave functions, too.

# Opaque Branes

- 5d theory with charged matter living on a brane
- At the one-loop level, the loops of charged matter will renormalize the gauge coupling
- These loops are **log** divergent, and contribute only on the brane:

$$-\delta(x_5) \frac{\alpha_5}{4\pi} \log \left[ \frac{\Lambda}{\mu} \right] \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}$$

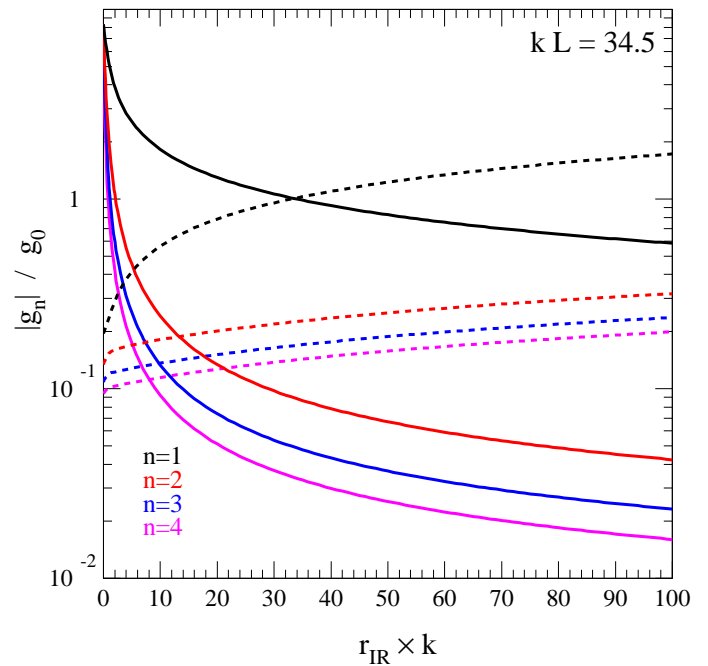
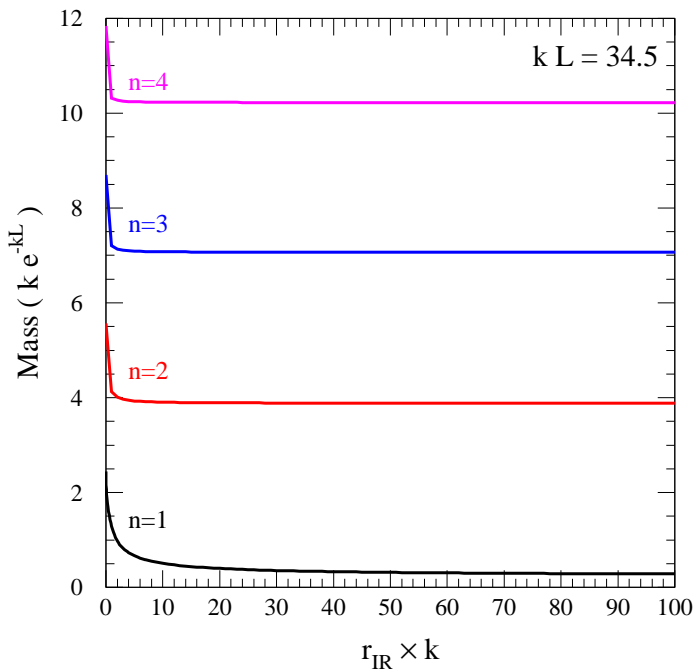
- The appearance of the divergence signals sensitivity to the UV physics
- $r_c$  cannot be predicted in terms of other quantities
- In that sense, it should be regarded as tree level
- This *does* tell us how  $r_c$  runs with  $\mu$
- We can self-consistently assume  $r_c$  is small at some scale. This amounts to an assumption about the nature of the UV completion

- Wave functions take the same form; new eigenmass

equation: [Davoudiasl, Hewett, Rizzo; Carena, Ponton, TT, Wagner]

$$\frac{\mathbf{J}_0\left(\frac{m_n}{k}\right) + m_n r_{UV} \mathbf{J}_1\left(\frac{m_n}{k}\right)}{\mathbf{Y}_0\left(\frac{m_n}{k}\right) + m_n r_{UV} \mathbf{Y}_1\left(\frac{m_n}{k}\right)} = \frac{\mathbf{J}_0\left(\frac{m_n}{k} e^{kL}\right) - m_n r_{IR} e^{kL} \mathbf{J}_1\left(\frac{m_n}{k} e^{kL}\right)}{\mathbf{Y}_0\left(\frac{m_n}{k} e^{kL}\right) - m_n r_{IR} e^{kL} \mathbf{Y}_1\left(\frac{m_n}{k} e^{kL}\right)},$$

- Note that  $r_{IR}$  is enhanced by the warp factor.



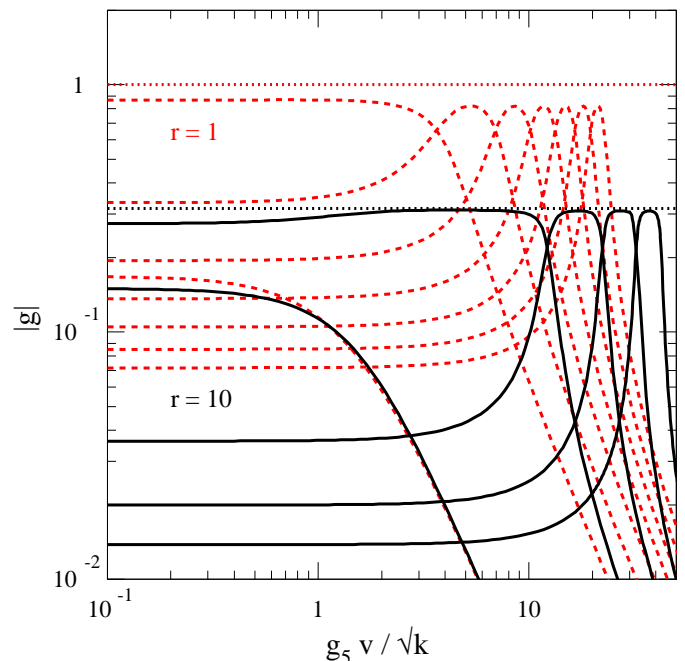
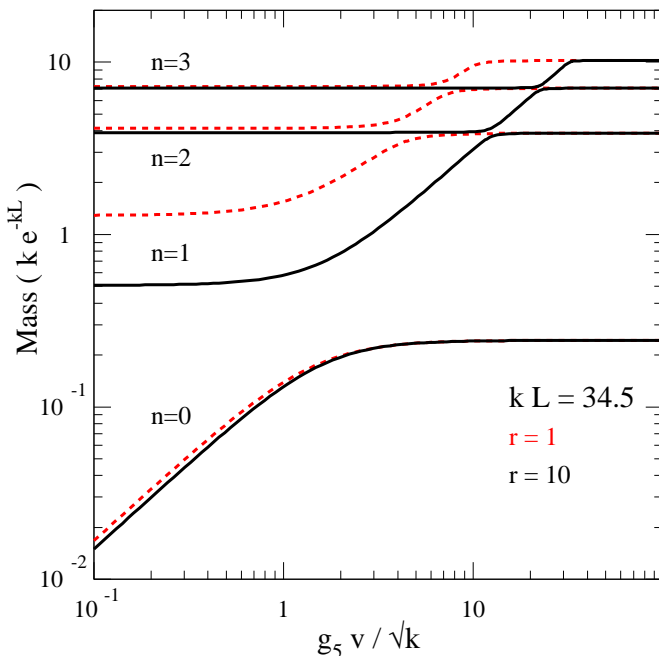
- Even for moderate  $r_{IR}$ , KK masses much smaller, couplings weaker.
- Again, there is a zero mode with flat wave function.

# Localized EWSB

- Our theory has the Higgs confined to a brane.
- If its VEV is large compared to the curvature, this will distort the wave functions.

$$\frac{\mathbf{J}_0\left(\frac{m_n}{k}\right) + m_n r_{UV} \mathbf{J}_1\left(\frac{m_n}{k}\right)}{\mathbf{Y}_0\left(\frac{m_n}{k}\right) + m_n r_{UV} \mathbf{Y}_1\left(\frac{m_n}{k}\right)} = \frac{\mathbf{J}_0\left(\frac{m_n}{k} e^{kL}\right) - m_n \tilde{r}_{IR} e^{kL} \mathbf{J}_1\left(\frac{m_n}{k} e^{kL}\right)}{\mathbf{Y}_0\left(\frac{m_n}{k} e^{kL}\right) - m_n \tilde{r}_{IR} e^{kL} \mathbf{Y}_1\left(\frac{m_n}{k} e^{kL}\right)},$$

- $\tilde{r}_{IR} \equiv r_{IR} - g_5^2 v^2 e^{-2kL} / m_n^2$
- For  $v \gg k$ , the  $W$  and  $Z$  are really KK modes.



- Large  $v$  is more like boundary condition EWSB.

## KK Mode Mixing

- We will restrict ourselves to  $v \ll k$ .
- Even in that more benign case, there are potentially large corrections to precision observables.
- We work in the  $v = 0$  basis and treat  $v/k$  as a perturbation.
- The wave function distortion manifests as mixing between the zero mode and the higher KK modes.
- The KK mode coupling to the Higgs is enhanced by  $kL \sim 30$  by the KK wave functions.
- This large factor exaggerates corrections from the mixing.
- Work in progress by **ADS** compensates with an  $SU(2)_R$  custodial gauge symmetry.
- We will stick with the SM gauge group and live with the mixing.

# Bulk Fermions

- Bulk fermions have odd bulk 'masses' coming from the curvature.
- There is also an 'odd' mass parameter allowed, which we write as  $M = ck$ .

$$\left[ \partial_y - k \left( \frac{1}{2} \mp c \right) \right] f_{L,R}^n(y) = \pm e^{kL} m_n f_{R,L}^n(y)$$

- The zero modes have exponential profiles around either the UV ( $c > 1/2$ ) or the IR brane ( $c < 1/2$ ).
- $c = 1/2$  results in a flat profile.
- This can be understood as a **Kaplan fermion**, because the odd mass is a thin domain wall.
- The localized Higgs VEV will generally distort fermion wave functions.
- However, for the light fermions this effect will be small.
- Higher KK modes are Bessel functions.

## EW Constraints on RS

- The usual statement is that RS with bulk gauge fields requires  $M \gtrsim 27 \text{ TeV}$  because of EW data.
- $\Delta\rho$  from mixing with KK modes:
  - The localized Higgs mixes the zero mode with the higher KK modes.
  - This upsets the relations between  $M_W$  and  $M_Z$  and  $\sin^2 \theta_W$  as measured in fermion interactions.
  - This effectively looks like a contribution to  $T$ :
$$\alpha\Delta T = -\tilde{v}^2 \tilde{G}_0^B(L, L) + \dots$$
  - $\tilde{G}_p(y_1, y_2)$  is the 5d propagator at 4d momentum  $p$  from  $y_1$  to  $y_2$ , with the zero mode piece ( $\sim 1/p^2$ ) removed.
  - This is a convenient way to sum over KK modes.
  - For KK masses  $\sim \text{TeV}$ ,  $\Delta T \sim .4$ .
- We will explore (small)  $r_{IR}$  to address this.

# Compositeness

- Exchange of KK gauge bosons:

- These induce 4-fermion operators, like parameterizations of compositeness:

$$-\frac{1}{2} [G_{ff}^3 J_\mu^3 J_3^\mu + G_{ff}^B J_\mu^Y J_Y^\mu] - G_{ff}^3 J_\mu^+ J_-^\mu$$

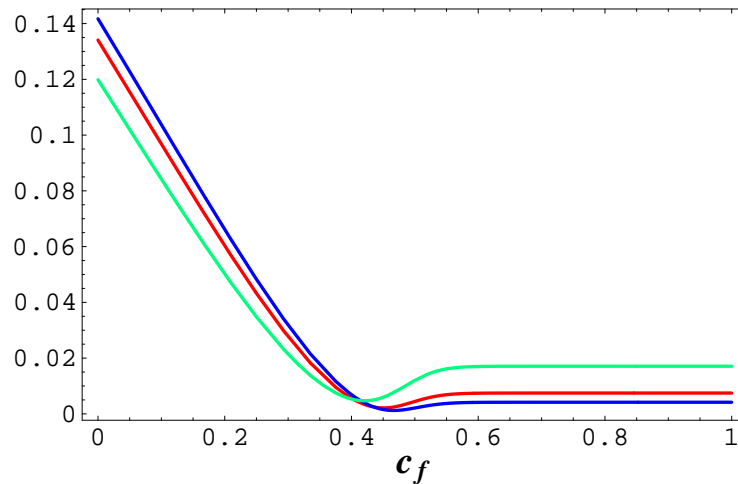
- The  $J_\mu$  are the gauge currents.

- $G_{ff}$  describe the exchange of KK modes:

$$G_{ff} \equiv \int_0^L dy dy' |f(y)|^2 \tilde{G}_0(y, y') |f(y')|^2$$

- These depend on the fermion  $c$  and gauge  $r$ 's.

- $G_{ff}/G_{LL}$ :



- $r_{IR} = 2, r_{UV} = -10, 0, 10$ .

- Non-oblique; small for  $c \sim 1/2$ .

# S Parameter

- Mixing between KK gauge bosons and zero modes

also corrects fermion interactions:

$$-\frac{e}{s c} Z_\mu \left[ (c^2 + \tilde{v}^2 G_f^3) J_3^\mu - (s^2 + \tilde{v}^2 G_f^B) J_Y^\mu \right]$$

- $G_f$  describe the exchange of KK modes:

$$G_f \equiv \int_0^L dy \tilde{G}_0(L, y) |f(y')|^2$$

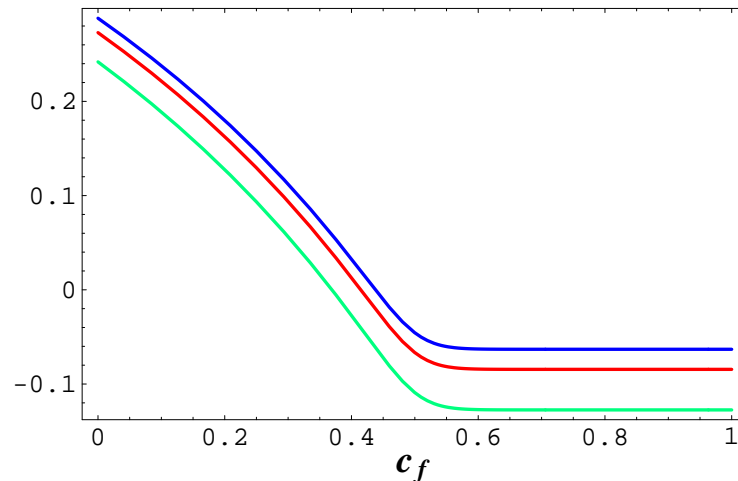
- Corrections to  $\Delta T$  and  $\Delta S$ :

$$\alpha \Delta S = 4\tilde{v}^2 [s^2 G_f^3 + c^2 G_f^B]$$

$$\alpha \Delta T = 2\tilde{v}^2 G_f^B + \dots$$

- $G_f$  depend on the fermion  $c$  and gauge  $r$ 's.

- $G_f/G_{LL}$ :



- $r_{IR} = 2, r_{UV} = -10, 0, 10$ .

- FCNC's for non-universal  $c$ 's.

## Top Yukawa Coupling

- The results suggest  $c \gtrsim 0.5$  is a good choice.
- Having fermions away from the IR brane naturally explains why their masses are  $m_f \ll v$ .
- However, for top we need  $m_t \sim v$ .
- The relevant quantity is the wave function at the Higgs:

$$a_f = \sqrt{(1 - 2c_f)kL} \quad (c_f < 1/2)$$

- Top Yukawa:

$$y_t = a_{t_L} a_{t_R} \frac{y_5}{L} \tilde{v}$$

- This is perturbative for  $c$  slightly less than 0.5.
- Anything we do to  $t_L$  will also influence  $b_L$ .
- If  $b$  has a different  $c$  than  $s$  or  $d$ , FCNCs result.
- To avoid FCNC's, we move all fermions to  $c \sim 0.3$ .

## Top KK Modes

- Loops of KK tops will correct  $S$  and  $T$ .
- The largest corrections are from loops of 2 KK modes:

$$\Delta T \sim \left( \frac{2kL}{a_L a_R} \right)^4 \left( \frac{m_t}{m_1} \right)^2 \left[ \frac{N_c}{16\pi s^2 c^2} \left( \frac{m_t}{m_Z} \right)^2 \right]$$

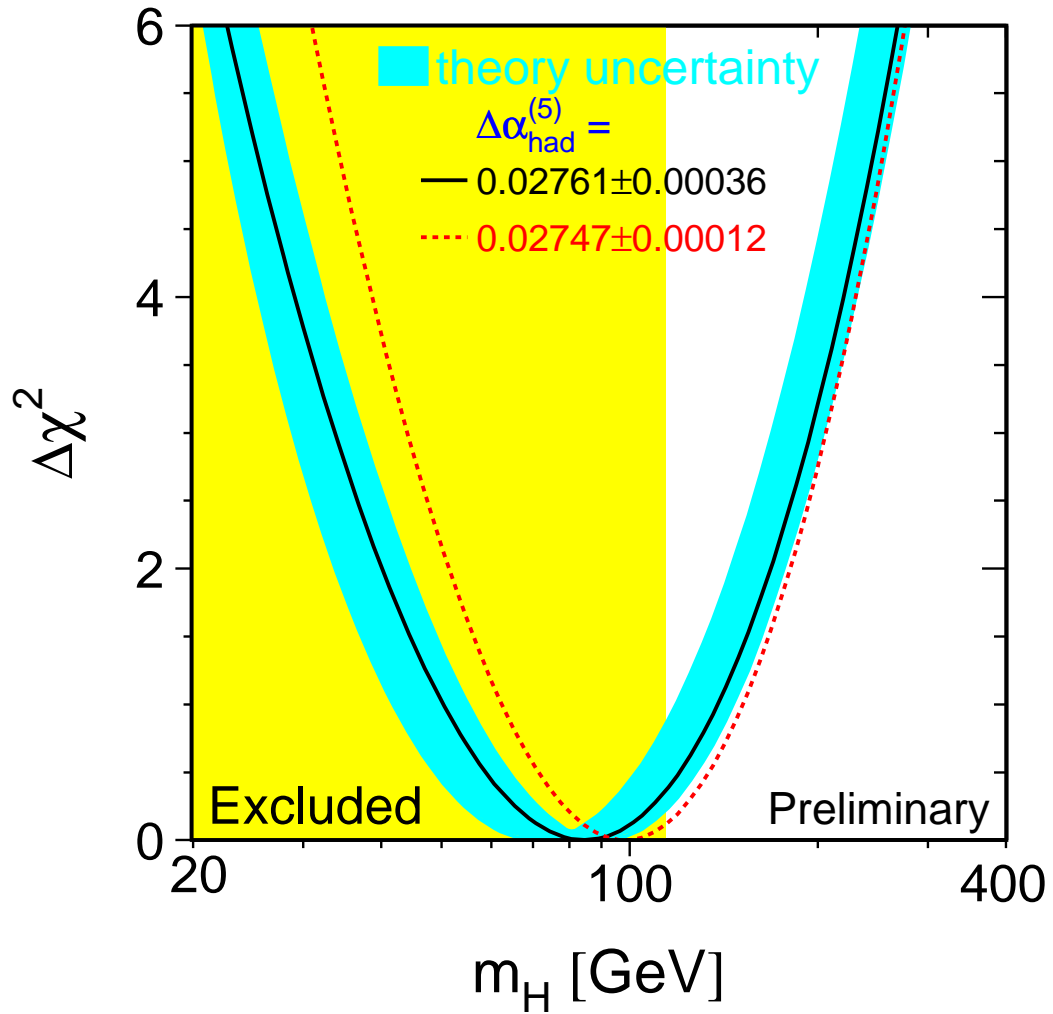
- To make this small, we take  $c_{t_R} \sim -5$ .
- Also, a  $t_R$  zero mode and  $t_L$  KK modes:

$$\Delta T \sim \frac{(2kL)^2}{3a_L^4} \left( \frac{m_t}{m_1} \right)^2 \left[ \frac{N_c}{16\pi s^2 c^2} \left( \frac{m_t}{m_Z} \right)^2 \right]$$

- This motivates  $c \sim 0.3$ .
- A related issue is  $XY$  KK modes in loops. We make these small by breaking the GUT on the IR brane.
- For these parameter choices, the tree level gauge mixing effects are dominant.
- The non-oblique corrections are tiny, and  $U \sim 0$ .
- Thus, an  $S$ - $T$  analysis is appropriate.

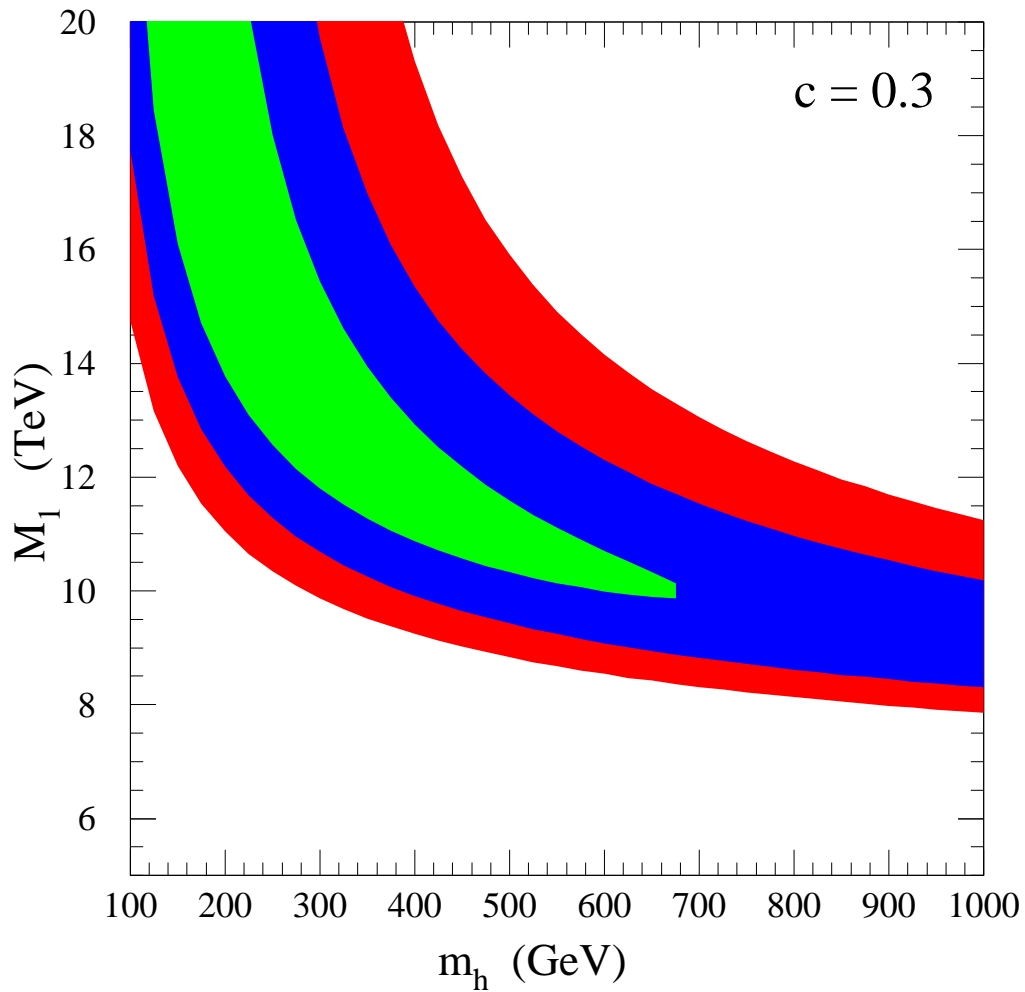
## EW Fit

- Global SM fit, in terms of  $m_H$ :



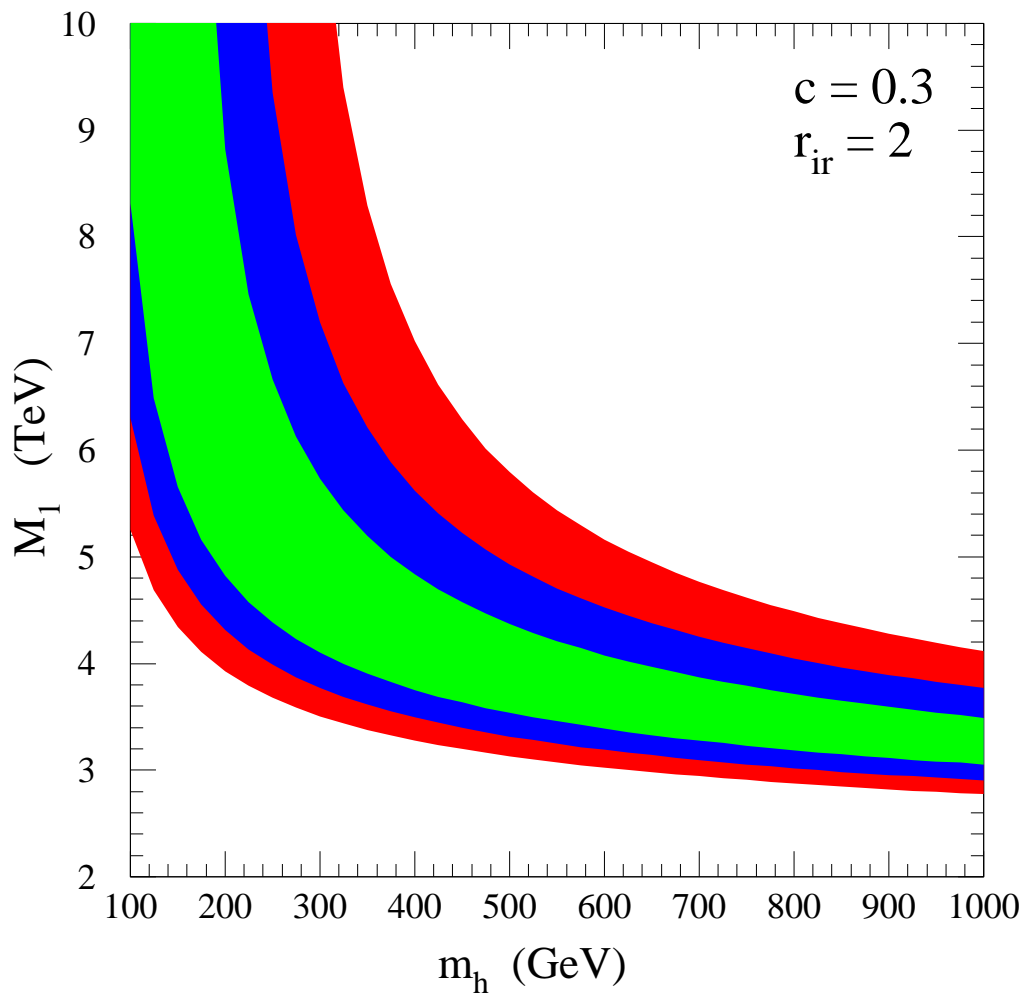
- Generally, the fit prefers a small Higgs mass.
- The large  $\Delta T$  contributions in RS will alter this, in a 'conspiracy theory' way. [Peskin, Wells]

- Putting everything together, we derive bounds on the KK masses for given  $m_H$ .
- No  $r_{IR}$  brane terms:



- 10 TeV masses for moderate  $m_H$ .
- Might be visible as compositeness at the LHC.

- $r_{IR} = 2$ :



- Few TeV masses for moderate  $m_H$ .
- Certainly visible at LHC.
- Note that large  $m_H$  is not as problematic as for a standard GUT.

## Linear Collider?

- The large KK masses indicate that at all but the highest energies, KK modes will not be produced there.
- Nonetheless, there is still important physics LC can do!
- Carefully study the Higgs : signs of compositeness!
- Low IR brane cut-off indicates that UV physics is most important for the Higgs.
- Higher dimensional operators involving Higgs are only suppressed by a few TeV.
- Top EW interactions will show deviations due to mixing of top with its KK modes.
- Even with modest energy, LC can be sensitive to off-shell  $Z'$ -like effects to masses of several TeV.

[ Hewett, Rizzo]

# Outlook

- RS with gauge fields in the bulk is an interesting theory which addresses the Hierarchy problem in a novel way.
- Unification of couplings is possible.
- Brane kinetic terms control the **masses** and **couplings** of the KK towers.
- The EW fit favors a **heavy** Higgs boson, with large  $m_H$  balancing  $\Delta T$  from the KK mode effects.
- Radion mixing?!
- KK mode masses of a few TeV are possible for modest  $r_{IR}$ .
- Phenomenology includes KK mode gauge bosons in complete GUT multiplets, KK fermions, etc.
- LC can study this theory by measuring a number of indirect properties, even if the masses are above its operating energy.