QCD Calculations at NLO: adding showers

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Work with

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M. Krämer and D. E. Soper, Phys. Rev. D **66**, 054017 (2002) [arXiv:hep-ph/0204113]; [arXiv:hep-ph/0306222]; D. E. Soper [arXiv:hep-ph/0306268]; S. Frixione and B. R. Webber, JHEP **2606**, 029 (2002) [arXiv:hep-ph/0204244]; S. Frixione, P Nason, and B. R. Webber, [arXiv:hep-ph/0305252].

Points to take home

- Parton showers can be added.

- The basic idea is very simple.

- You should add radiation of a soft gluon.

- The primary splittings in the showers plus the soft gluon radiation are the responsibility of the NLO author.

Adding showers

• Produce realistic final states f. (Hadrons, but for the present just a parton shower down to some minimum virtuality.)

$$\mathcal{I} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} w_n \, \mathcal{S}(f_n).$$

• For an infrared safe observable, the result $\mathcal{I}$ should have a perturbative expansion,

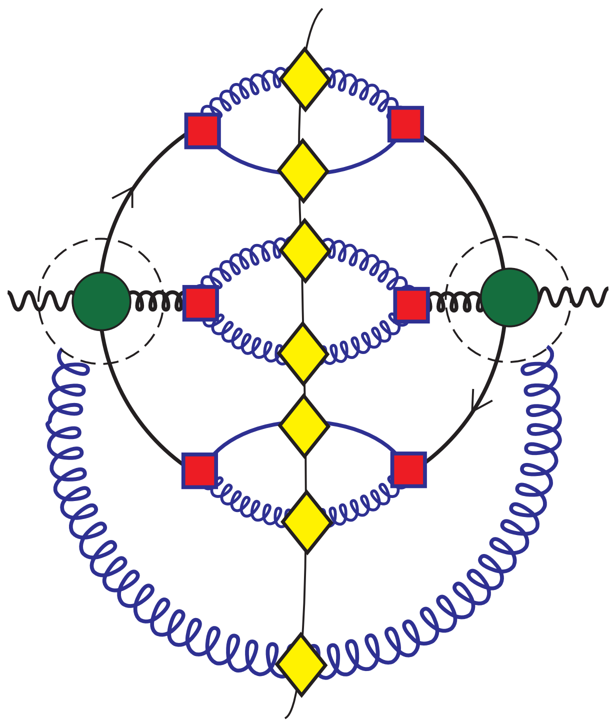
$$\mathcal{I} = C_0 \, \alpha_s^B + C_1 \, \alpha_s^{B+1} + C_2 \, \alpha_s^{B+2} + \cdots .$$

• Keep C_0 and C_1 exactly the same.

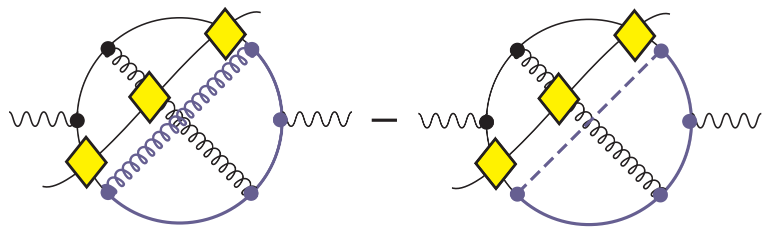
• *Cf.* Frixione, Nason, & Webber.

Structure of the calculation

From α_s^B diagrams.
Each parton splits.
A soft gluon is radiated.
Partons $\rightarrow$ showers.

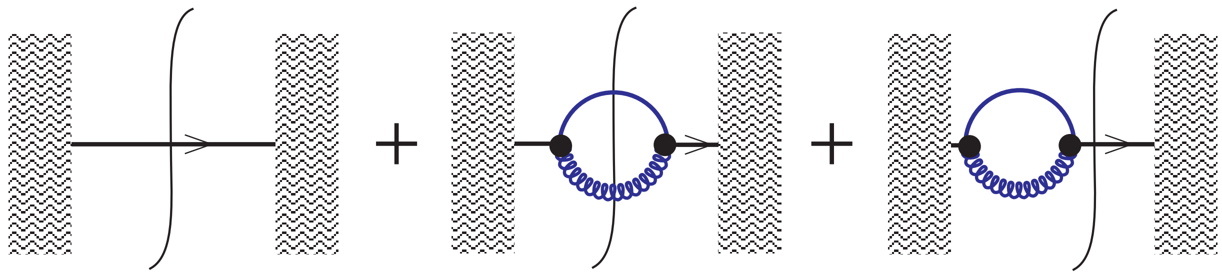


From α_s^{B+1} diagrams with subtractions.



Collinear singularities

Perturbative calculation in the Coulomb gauge.

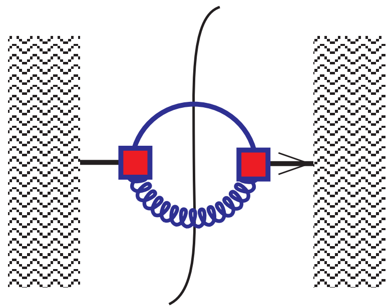


$$\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] =$$
$$\int \frac{d\vec{q}}{2|\vec{q}|}\ \text{Tr}\Big\{ \not{q}\, R_0 + \int_0^\infty \frac{d\vec{q}^{\,2}}{\vec{q}^{\,2}} \int_0^1 dx \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}$$
$$\times \left[\frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\vec{q}^{\,2}, x, \phi) R(\vec{q}^{\,2}, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\vec{q}^{\,2}, x)\, \not{q}\, R_0 \right] \Big\}.$$

- $R(\vec{q}^{\,2}, x, \phi)$ and R_0 represent the rest of the graph, including the measurement function. $R(\vec{q}^{\,2}, x, \phi) \to R_0$ for $\vec{q}^{\,2} \to 0$.

- $\vec{q}^{\,2} \to 0$ singularities cancel.

Shower splitting.



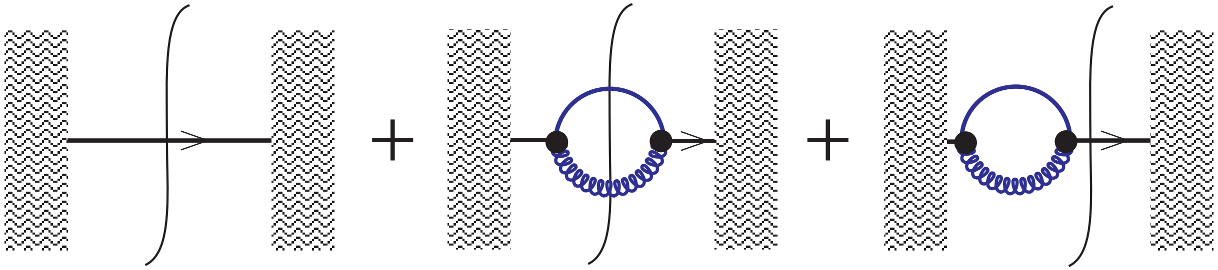
$$\mathcal{I}[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|}\,\mathrm{Tr}\Big\{\int_0^\infty \frac{d\bar{q}^{\,2}}{\bar{q}^{\,2}}\int_0^1 dx \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\,\frac{\alpha_s}{2\pi}\mathcal{M}_{g/q}(\bar{q}^{\,2},x,\phi)R(\bar{q}^{\,2},x,\phi)$$
$$\times\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^{\,2}}{\bar{l}^{\,2}}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\mathcal{P}_{g/q}(\bar{l}^{\,2},z)\right)\Big\}.$$

- Splitting with the exact Coulomb gauge $\mathcal{M}$.
- Sudakov suppression with the exact Coulomb gauge $\mathcal{P}$.

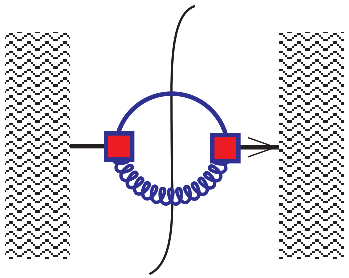
Subtraction to multiplication theorem

$$\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] = \mathcal{I}[\text{shower}] \times \Big(1 + \mathcal{O}(\alpha_s^2)\Big)\,.$$

That is



is equivalent to



Proof step 0. Add and subtract:

$$\mathcal{I}[\text{shower}] = \mathcal{I}_1[\text{shower}] + \mathcal{I}_2[\text{shower}]$$

with

$$\begin{aligned}
&\mathcal{I}_1[\text{shower}] \\
&= \int \frac{d\vec{q}}{2|\vec{q}|}\ \mathrm{Tr}\Big\{\int_0^\infty \frac{d\bar{q}^{\,2}}{\bar{q}^{\,2}} \int_0^1 dx \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}\ \exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\ \frac{\alpha_s}{2\pi}\mathcal{P}_{g/q}(\bar{l}^2, z)\right) \\
&\quad \times \left[\frac{\alpha_s}{2\pi}\mathcal{M}_{g/q}(\bar{q}^{\,2}, x, \phi)\, R(\bar{q}^{\,2}, x, \phi) - \frac{\alpha_s}{2\pi}\mathcal{P}_{g/q}(\bar{q}^{\,2}, x)\, \not{d}\, R_0\right]\Big\}
\end{aligned}$$

and

$$\begin{aligned}
&\mathcal{I}_2[\text{shower}] \\
&= \int \frac{d\vec{q}}{2|\vec{q}|}\ \mathrm{Tr}\Big\{\not{d}\, R_0 \int_0^\infty d\bar{q}^{\,2} \\
&\qquad \times \frac{1}{\bar{q}^{\,2}}\int_0^1 dx\ \frac{\alpha_s}{2\pi}\mathcal{P}_{g/q}(\bar{q}^2, x)\ \exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\ \frac{\alpha_s}{2\pi}\mathcal{P}_{g/q}(\bar{l}^2, z)\right)\Big\}.
\end{aligned}$$

Proof step 1. Expand $\mathcal{I}_1$[shower]:

$$
\begin{aligned}
&\mathcal{I}_1[\text{shower}] \\
&= \int \frac{d\vec{q}}{2|\vec{q}|}\ \mathrm{Tr}\Big\{ \int_0^\infty \frac{d\bar{q}^{\,2}}{\bar{q}^{\,2}} \int_0^1 dx \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}\ \exp\left(-\int_{\bar{q}^{\,2}}^{\infty} \frac{d\bar{l}^{\,2}}{\bar{l}^{\,2}} \int_0^1 dz\ \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^{\,2}, z) \right) \\
&\quad \times \left[\frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^{\,2}, x, \phi)\, R(\bar{q}^{\,2}, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^{\,2}, x)\, \not{d}\, R_0 \right] \Big\} \\
&= \int \frac{d\vec{q}}{2|\vec{q}|}\ \mathrm{Tr}\Big\{ \int_0^\infty \frac{d\bar{q}^{\,2}}{\bar{q}^{\,2}} \int_0^1 dx \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \times 1 \\
&\quad \times \left[\frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^{\,2}, x, \phi)\, R(\bar{q}^{\,2}, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^{\,2}, x)\, \not{d}\, R_0 \right] \Big\} + \mathcal{O}(\alpha_s^2 \times R) \\
&= \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] + \mathcal{O}(\alpha_s^2 \times R)
\end{aligned}
$$

Proof step 2. Calculate $\mathcal{I}_2[\text{shower}]$:

$$
\begin{aligned}
&\mathcal{I}_2[\text{shower}] \\
&\quad = \int \frac{d\vec{q}}{2|\vec{q}|}\ \text{Tr}\Big\{\not{q}\, R_0 \int_0^\infty d\bar{q}^{\,2} \\
&\qquad\qquad \times \frac{1}{\bar{q}^{\,2}} \int_0^1 dx\ \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^{\,2}, x)\ \ \exp\left(- \int_{\bar{q}^{\,2}}^\infty \frac{d\bar{l}^{\,2}}{\bar{l}^{\,2}} \int_0^1 dz\ \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^{\,2}, z) \right) \Big\} \\
&\quad = \int \frac{d\vec{q}}{2|\vec{q}|}\ \text{Tr}\Big\{\not{q}\, R_0 \Big\}. \\
&\quad = \mathcal{I}[\text{Born}].
\end{aligned}
$$

That is

$$
\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] = \mathcal{I}[\text{shower}] \times \left(1 + \mathcal{O}(\alpha_s^2)\right).
$$

You could use your favorite $\mathcal{M}'$ & $\mathcal{P}'$

$$\mathcal{I}'[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|}\ \mathrm{Tr}\Big\{\int_0^\infty \frac{d\bar{q}^{\,2}}{\bar{q}^{\,2}} \int_0^1 dx \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}\ \frac{\alpha_s}{2\pi} \mathcal{M}'_{g/q}(\bar{q}^{\,2}, x, \phi) R(\bar{q}^{\,2}, x, \phi)$$
$$\times \exp\left(- \int_{\bar{q}^2}^\infty \frac{d\bar{l}^{\,2}}{\bar{l}^{\,2}} \int_0^1 dz\ \frac{\alpha_s}{2\pi} \mathcal{P}'_{g/q}(\bar{l}^{\,2}, z)\right)\Big\}.$$

Then

$$\mathcal{I}'[\text{shower}] =$$
$$\int \frac{d\vec{q}}{2|\vec{q}|}\ \mathrm{Tr}\Big\{\not{q}\, R_0 + \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}$$
$$\times \left[\frac{\alpha_s}{2\pi} \mathcal{M}'_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}'_{g/q}(\bar{q}^2, x)\, \not{q}\, R_0\right]\Big\} + \mathcal{O}(\alpha_s^2 \times R)$$
$$= \mathcal{I}[\text{Born}] + \mathcal{I}'[\text{real}] + \mathcal{I}'[\text{virtual}].$$

What you get then

$$\mathcal{I}'[\text{shower}] = \mathcal{I}[\text{Born}] + \mathcal{I}'[\text{real}] + \mathcal{I}'[\text{virtual}].$$

gives

$$\begin{aligned} & \mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] \\ & \quad = \mathcal{I}'[\text{shower}] + (\mathcal{I}[\text{real}] - \mathcal{I}'[\text{real}]) + (\mathcal{I}[\text{virtual}] - \mathcal{I}'[\text{virtual}]). \end{aligned}$$

The $\mathcal{M}'$ and $\mathcal{P}'$ functions act as subtractions for $\mathcal{M}$ and $\mathcal{P}$.

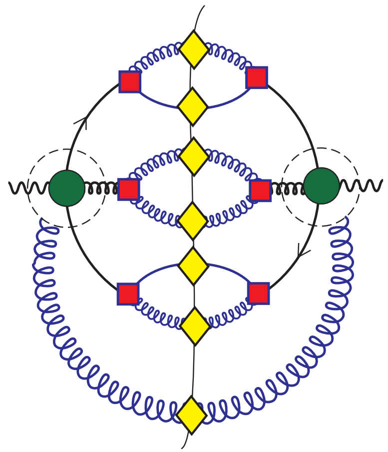
As long as they have the right $\vec{q}^{\,2} \to 0$ singularities, they cancel the singularities of $\mathcal{M}$ and $\mathcal{P}$.

Soft radiation

Three jets make an antenna.
Radiate one gluon.
Use eikonal approximation,

$$F_{ij}(\hat{l};\hat{q}_1,\hat{q}_2,\hat{q}_3).$$

Part of F_{ii} in jets.

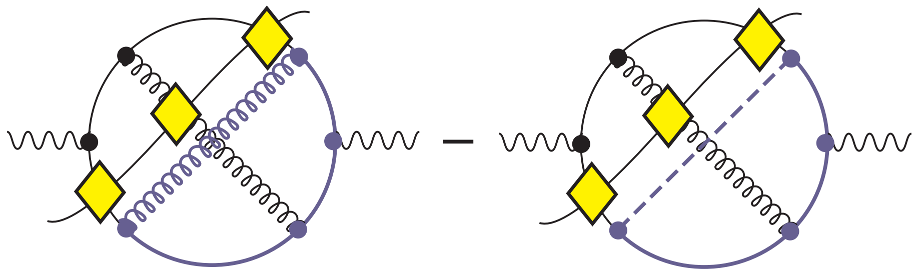


$$\begin{aligned}\mathcal{I}[\text{radiate}] \;=\;& \int d\vec{q}_1 \int d\vec{q}_2 \int d\vec{q}_3\ \delta\Big(\sum \vec{q}_i\Big) \int_0^{M_{\rm soft}} \frac{d|\vec{l}\,|}{|\vec{l}\,|} \int \frac{d^2\hat{l}}{4\pi} \\ &\times \sum_{ij} \frac{\alpha_s}{\pi} F_{ij}(\hat{l};\hat{q}_1,\hat{q}_2,\hat{q}_3)\, R_{ij}(\vec{l};\vec{q}_1,\vec{q}_2,\vec{q}_3) \\ &\times \exp\left(- \int_{|\vec{l}|}^{M_{\rm soft}} \frac{d|\vec{l}^{\,\prime}\,|}{|\vec{l}^{\,\prime}\,|} \int \frac{d^2\hat{l}^{\prime}}{4\pi} \sum_{i'j'} \frac{\alpha_s}{\pi} F_{i'j'}(\hat{l}^{\prime};\hat{q}_1,\hat{q}_2,\hat{q}_3) \right) .\end{aligned}$$

Left over contributions

There remain α_s^{B+1} diagrams with subtractions.

For example,



A test

Calculate $d\sigma/dt$ at thrust $t = 0.86$ ($\sqrt{s} = M_Z$, $\mu = \sqrt{s}/6$).

$$R = \frac{\text{(NLO-shower)} - \text{NLO}}{\text{NLO}}.$$

If calculation is correct,

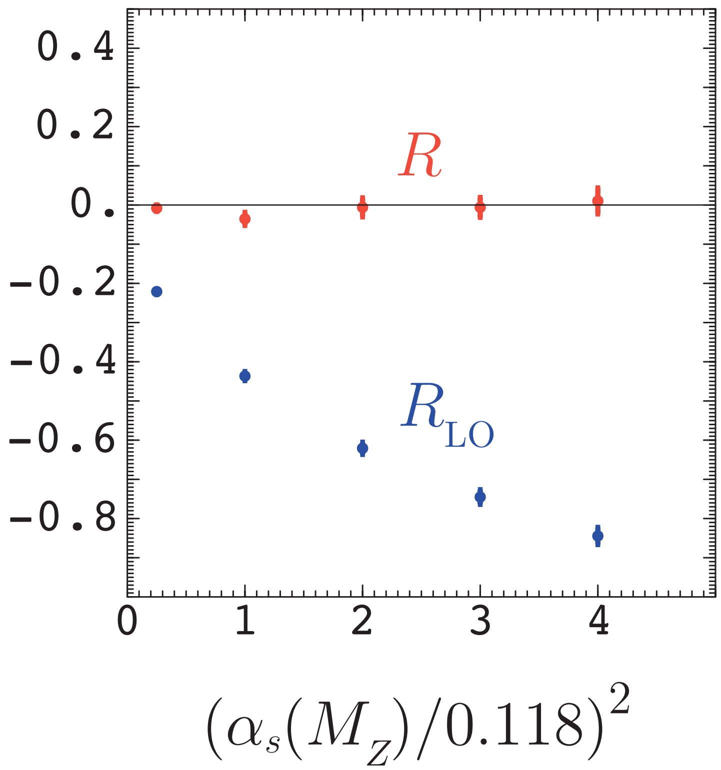
$$R = \frac{(C_0\alpha_s^B + C_1\alpha_s^{B+1} + C_2\alpha_s^{B+2} + \cdots) - (C_0\alpha_s^B + C_1\alpha_s^{B+1})}{C_0\alpha_s^B + C_1\alpha_s^{B+1}}$$

so

$$R = \frac{C_2}{C_0}\,\alpha_s^2 + \cdots.$$

So plot R versus $\alpha_s(M_Z)^2$.

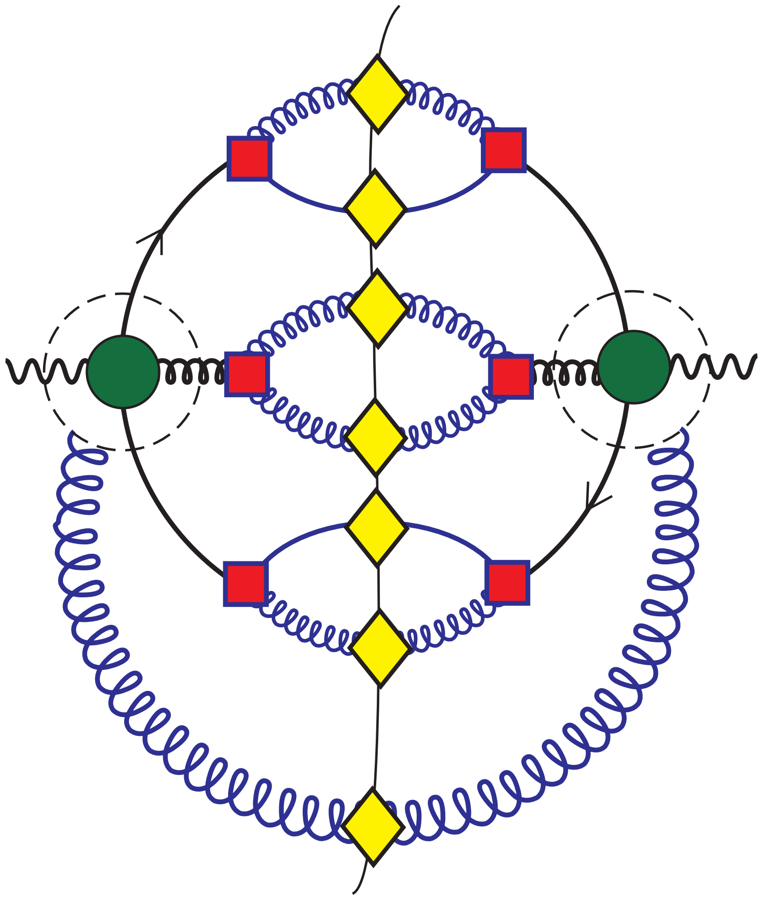
The result



- $R_{\rm LO}$ is the same without the order α_s^{B+1} corrections.

My guess about the future

- NLO calculations will include showers.
- NLO authors will be responsible for
  - primary splittings
  - soft gluon
- The rest will come from Pythia, Herwig, Ariadne . . .
- Users will mix and match.



Points to take home

- Parton showers can be added to an NLO calculation.

- The basic idea is very simple.

- You should add radiation of a soft gluon.

- The primary splittings in the showers plus the soft gluon radiation are the responsibility of the NLO author.