In special relativity, spacetime interval ds^2 is invariant:

$$ds^2 = (c\,dt)^2 - \left(dx^2 + dy^2 + dz^2\right)$$

In GR:

$$ds^2 = (c\,dt)^2 - R(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

$R(t)$: *scale parameter*,
K: spacetime curvature:

- $K = 0$. Flat spacetime.
- $K = +1$. Positive curvature.
- $K = -1$. Negative curvature.

Einstein field equations of general relativity:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$T_{\mu\nu}$: stress energy tensor,
$G_{\mu\nu}$: spacetime curvature, and
Λ: the (infamous) cosmological constant

Connection to observations:

$$R(t) = R_o/(1+z)\ ,$$

$$\dot{R} = HR\ ,$$

(Hubble: $v = Hd$), where $\dot{f} = df/dt$,
z: redshift; $z = \Delta\lambda/\lambda$,
$H_o \approx 70$ (km/s)/Mpc

Friedmann equation for the scale parameter R:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_{\rm tot} - \frac{Kc^2}{R^2}$$

$\rho_{\rm tot} = \rho_m + \rho_r + \rho_v$: total energy density,
$\Omega \equiv \rho/\rho_c$, $\rho_c = 3H_o^2/(8\pi G)$

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho_{\text{tot}} - \frac{Kc^2}{R^2}$$

We can gain significant intuition about the Friedmann equation by showing its equivalence to a more familiar model. Let us assume that ρ is dominated by ordinary non-relativistic matter and that this density is uniform. We put a test mass m at the edge of a sphere of radius $R(t)$. Only the mass within the sphere, $M = 4\pi R^3\rho/3$, contributes to the gravitational force acting on m. (This Gauss's Law for gravity also holds in general relativity.) Hence, the potential energy of m is $-GMm/R$, and the total energy $E = E_K + U$ is

$$\frac{1}{2}m\dot{R}^2 - \frac{GMm}{R} = E\ .$$

Let $M = 4\pi R^3\rho/3$ and multiply through by $2/(mR^2)$, then the left-hand side is identical to that of the Friedmann equation. And the right-hand side, the total energy term, becomes $2E/(mR^2)$. Hence, we can make the identification $E = -Kc^2/(2m)$, or

$$E_K + U = E \propto -K\ .$$

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 - \left(\frac{8\pi G}{3}\right)\rho_{\rm tot} = -\frac{Kc^2}{R^2}$$

$$\frac{1}{2}m\dot{R}^2 - \frac{GMm}{R} = E = -Kc^2/(2m)\ .$$

- $K > 0 \Rightarrow E < 0 \Rightarrow$ "bound" ("closed"), R returns to 0.

- $K < 0 \Rightarrow E > 0 \Rightarrow$ "unbound" ("open"), R goes to infinity.

- $K = 0$ ("flat") $\Rightarrow E = 0 \Rightarrow R \to \infty$ and $\dot{R} \to 0$ as $t \to \infty$.

Current observations in the present epoch give the following best values:

- $\Omega_k \approx 0$ (cosmic μwave background observations, especially the most recent results from WMAP)

- $\Omega_r \approx 10^{-5}$

- $\Omega_m \approx 0.3$. All of normal (baryonic) matter is 0.05; the remainder is *dark matter*.

- $\Omega_v \approx 0.7$. This would be the contribution due to *dark energy*. Note that $\rho_v = \frac{\Lambda}{8\pi G}$.