

Lectures 7-8

§ Overview of differential equations:

- Sep. of variables → Linear, homog., 2nd-order ODEs:

$$0 = y'' + P(x)y' + Q(x)y \equiv \mathcal{L}y$$

- Examples: Legendre, Bessel, ... (Table 9.4)
- 2 Lin. inde. sol'ns: $y = c_1y_1 + c_2y_2$

§ Dirac delta function (§1.15)

$$\int_a^b f(x)\delta(x - x_0) dx = f(x_0), \quad a < x_0 < b$$

- ### § Green's function to find $y_p(x)$ for non-homog.:
- $\mathcal{L}y = F(x)$ (where $y = c_1y_1 + c_2y_2 + y_p$):

$$y_p(x) = \int G(x, x')F(x') dx'$$

Lectures 7-8 (contd.)

§ 2nd-order ODEs – general and self-adjoint forms:

- general: $0 = p_0(x)y'' + p_1(x)y' + p_2(x)y$
- self-adjoint: $0 = \frac{d}{dx} [p(x)y'] + q(x)y \equiv \mathcal{L}_s y$
- So, general \rightarrow self-adjoint $\Leftrightarrow p_0 = p, p_1 = p'_0$
- Examples: text Table 10.1
- self-adjoint is general (e.g. hw 10.1.1)

§ The Sturm-Liouville (eigenvalue) equation:

$$\mathcal{L}_s u(x) + \lambda w(x)u(x) = 0$$

§ Self-adjoint + BCs \Rightarrow Hermitian operator, \mathcal{L} :

$$\int_a^b u_i^* \mathcal{L} u_j dx = \int_a^b u_j (\mathcal{L} u_i)^* dx$$

- Real eigenvalues, λ_i
- orthogonal eigenfunctions, $u_i(x)$
- completeness (generalized Fourier series, Hilbert space)